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TITLE OF INVENTION: Hybrid Walsh Codes for CDMA

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# BACKGROUND OF THE INVENTION

## I. Field of the Invention

The present invention relates to CDMA (Code Division Multiple Access) cellular telephone and wireless data communications with data rates up to multiple T1 (1.544 Mbps) and higher (>100 Mbps), and to optical CDMA with data rates in the Gbps and higher ranges. Applications are mobile, point-to-point and satellite communication networks. More specifically the present invention relates to novel complex and generalized complex Walsh codes developed to replace current Walsh orthogonal CDMA channelization codes which are real Walsh codes.

## II. Description of the Related Art

Current CDMA art is represented by the recent work on multiple access for broadband wireless communications, the G3 (third generation CDMA) proposed standard candidates, the current IS-95 CDMA standard, the early Qualcomm patents, and the real Walsh technology. These are documented in "Multiple Access for Broadband Networks", IEEE Communications magazine July 2000 Vol. 38 No. 7, "Third Generation Mobile Systems in Europe", IEEE Personal Communications April 1998 Vol. 5 No. 2, IS-95/IS-95A, the IS-95/IS-95A, the 3G CDMA2000 and W-CDMA, and the listed patents.

Current art using Walsh orthogonal CDMA channelization codes is represented by the scenario described in the following with the aid of equations **(1)** and FIG **1,2,3,4**. This scenario considers CDMA communications spread over a common frequency band for each of the communication channels with each channel defined by a CDMA code. These CDMA communications channels for each of the users are defined by assigning a unique Walsh orthogonal spreading codes to each user. The Walsh code for each user spreads the user data symbols over the common frequency band. These Walsh encoded user signals are summed and re-spread over the same frequency band by long and short pseudo-noise PN codes, to generate the CDMA communications signal which is modulated and transmitted. The communications link consists of a transmitter, propagation path, and receiver, as well as interfaces and control.

It is assumed that the communication link is in the communications mode with all of the users communicating at the same symbol rate and the synchronization is sufficiently accurate and robust to support this communications mode. In addition, the possible power differences between the users is assumed to be incorporated in the data symbol amplitudes prior to the CDMA encoding in the CDMA transmitter, and the power is uniformly spread over the wideband by proper selection of the CDMA pulse waveform.

Transmitter equations **(1)** describe a representative real Walsh CDMA encoding for the transmitter in FIG. **1**. It is assumed that there are N Walsh code vectors W(u) each of length N chips **1**. The code vector is presented by a 1xN N-chip row vector W(u)=[W(u,1),…,W(u,N)] where W(u,n) is chip n of code u. The code vectors are the row vectors of the Walsh matrix W. Walsh code chip n of code vector u has the possible values W(u,n)= +/-1. Each user is assigned a unique Walsh code which

allows the code vectors to be designated by the user symbols $u=0,1,\ldots,N-1$ for N Walsh codes. User data symbols **2** are the set of complex symbols $\{Z(u),\ u=0,,1,\ldots,N-1)$ and the set of real symbols $(R(u_R),\ I(u_I),\ \ u_R,\ u_I=0,1,\ldots,N-1\}$ where Z is a complex symbol and R,I are real symbols assigned to the real, imaginary communications axis. Examples of complex user symbols are QPSK and OQPSK encoded data corresponding to 4-phase and offset 4-phase symbol coding. Examples of real user symbols are PSK and DPSK encoded data corresponding to 2-phase and differential 2-phase symbol coding. Although not considered in this example, it is possible to use combinations of both complex and real data symbols.

Current real Walsh CDMA encoding for transmitter **(1)**

**1** Walsh codes

W = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors
= [ W(u) ] matrix of row vectors W(u)
= [ W(u,n) ] matrix of elements W(u,n)

W(u) = Walsh code vector u for u=0,1,…,N-1
= [ W(u,0), W(u,1), …, W(u,N-1) ]
= 1xN row vector of chips W(u,0),….,W(u,N-1)

W(u,n) = Walsh code u chip n
= +/+1 possible values

**2** Data symbols

Z(u) = Complex data symbol for user u

$R(u_R)$ = Real data symbol for user $u_R$ assigned to the Real axis of the CDMA signal

$I(u_I)$ = Real data symbol for user $u_I$ assigned to th Imaginary axis of the CDMA signal

**3** Walsh encoded data

Complex data symbols

$Z(u,n) = Z(u)\ \mathrm{sgn}\{\ W(u,n)\}$

$=$ User u chip n Walsh encoded complex data

Real data symbols

$R(u_R,n) = R(u_R)\ \mathrm{sgn}\{\ W(u_R,n)\ \}$

$=$ User $u_R$ chip n Walsh encoded real data

$I(u_I,n) = R(u_R)\ \mathrm{sgn}\{\ W(u_R,n)\ \}$

$=$ User $u_I$ chip n Walsh encoded real data

where $\mathrm{sgn}\{\ (o)\ \}$ = Algeraic sign of "(o)"

**4** PN scrambling

$P_2(n)\ , P_{R2}(n), P_{I2}(n)$ = Chip n of long PN codes

$P_R(n)$ = Chip n of short PN code for real axis

$P_I(n)$ = Chip n of short PN code for imaginary axis

Complex data symbols:

$Z(n)$ = PN scrambled Walsh encoded data chips after summing over the users

$$= \sum_u \mathbf{Z(u,n)P_2(n)[P_R(n)+j\,P_I(n)]}$$

$$= \sum_u \mathbf{Z(u,n)}\mathrm{sgn}\{P_2(n)\}[\mathrm{sgn}\{P_R(n)\}+j\,\mathrm{sgn}\{P_I(n)\}]$$

$=$ Real Walsh CDMA encoded complex chips

Real data symbols:

$Z(n)\ =$

$$[\sum_{u_R}\mathbf{R(u_R,n)}\,\mathrm{sgn}\{P_{R2}(n)\}+j\sum_{u_I}\mathbf{I(u_I,n)}\mathrm{sgn}\{P_{I2}(n)\}\ \ [\mathrm{sgn}\{P_R(n)\}+j\ \mathrm{sgn}\{P_I(n)\}]$$

$=$ Real Walsh CDMA encoded real chips

User data is encoded by the Walsh CDMA codes **3.** Each of the user symbols $Z(u), R(u_R), I(u_I)$ is assigned a unique Walsh

code. $W(u), W(u_R), W(u_I)$. Walsh encoding of each user data symbol generates an N-chip sequence with each chip in the sequence consisting of the user data symbol with the sign of the corresponding Walsh code chip, which means each chip = [Data symbol] x [Sign of Walsh chip].

The Walsh encoded data symbols are summed and encoded with PN codes **4**. These long PN codes are 2-phase with each chip equal to +/-1 which means PN encoding consists of sign changes with each sign change corresponding to the sign of the PN chip. Short PN codes are complex with 2-phase codes along their real and imaginary axes. Encoding with a long PN means each chip of the summed Walsh encoded data symbols has a sign change when the corresponding long PN chip is -1, and remains unchanged for +1 values. This operation is described by a multiplication of each chip of the summed Walsh encoded data symbols with the sign of the PN chip. Purpose of the PN encoding is to provide scrambling of the summed Walsh encoded data symbols as well as isolation between groups of users and synchronization. PN encoding uses a long PN which is real followed by a short PN which is complex with real code components on the inphase and quadrature axes as shown in **4**.

Receiver equations **(2)** describe a representative Walsh CDMA decoding for the receiver in FIG. **3**. The receiver front end **5** provides estimates $\{\hat{Z}(n) = \hat{R}(n) + j\,\hat{I}(n)\}$ of the transmitted real Walsh CDMA encoded chips $\{Z(n)=R(n)+jI(n)\}$ for the complex and real data symbols. Orthogonality property **6** is expressed as a matrix product of the Walsh code chips or equivalently as a matrix product of the Walsh code chip numerical signs. Decoding algorithms **8** perform the inverse of the signal processing for the encoding in equations **(1)** to recover estimates $\{\hat{Z}(u)\}$ or $\{\hat{R}(u_R), \hat{I}(u_I)\}$ of the transmitter user symbols $\{Z(u)\}$ or $\{R(u_R), I(u_I)\}$ for the respective complex or real data symbols.

Current real Walsh CDMA decoding for receiver **(2)**

**5** Receiver front end provides estimates $\{\hat{Z}(n) = \hat{R}(n) + j\hat{I}(n)\}$ of the encoded transmitter chip symbols $\{Z(n)=R(n)+jI(n)\}$ for the complex and real data symbols

**6** Orthogonality property of Walsh NxN matrix W

$$\sum_n W(\hat{u},n)W(n,u) = \sum_n \mathrm{sign}\{W(\hat{u},n)\}\,\mathrm{sign}\{W(n,u)$$

$$= N\ \delta(\hat{u}, u)$$

where $\delta(\hat{u}, u)$ = Delta function of $\hat{u}$ and u

= 1 for $\hat{u} = u$

= 0 otherwise

**7** PN decoding property:

$$P_2(n)P_2(n) = \mathrm{sgn}\{P_2(n)\ \mathrm{sgn}\{P_2(n)\}$$

$$= 1$$

Decoding algorithm:

Complex data symbols

$$\hat{Z}(u) =$$

$$2^{-1}N^{-1}\sum_n \hat{Z}(n)[\mathrm{sign}\{P_2(n)\}\ [\mathrm{sgn}\{P_R(n)\} - j\,\mathrm{sign}\{P_I(n)\}]\ \mathrm{sgn}\{W(n,u)\}$$

= Receiver estimate of the transmitted complex data symbol Z(u)

Real data symbols

$$\hat{R}(u_R) =$$

$$\mathrm{Real}[\ 2^{-1}N^{-1}\sum_n \hat{Z}(n)[\mathrm{sgn}\{P_R(n)\} - j\,\mathrm{sgn}\{P_I(n)\}]\ \mathrm{sgn}\{P_2(n)\}\mathrm{sgn}\{W(n,u_R)\}$$

= Receiver estimate of the transmitted complex data symbol $R(u_R)$

$\hat{I}(u_I)$ =

$$\mathrm{Imag}[\ 2^{-1}N^{-1}\sum_n \hat{Z}(\mathbf{n})[\mathrm{sgn}\{P_R(n)\} - j\,\mathrm{sgn}\{P_I(n)\}]\ \mathrm{sgn}\{P_2(n)\}\mathrm{sgn}\{W(n,u_I)\}$$

= Receiver estimate of the transmitted complex data symbol $I(u_I)$

FIG. **1** CDMA transmitter block diagram is representative of a current CDMA transmitter which includes an implementation of the current real Walsh CDMA channelization encoding in equations **(1)**. This block diagram becomes a representative implementation of the CDMA transmitter which implements the complex Walsh CDMA encoding of this invention disclosure when the current real Walsh CDMA encoding **13** is replaced by the complex Walsh CDMA encoding. Signal processing starts with the stream of user input data words **9**. Frame processor **10** accepts these data words and performs the encoding and frame formatting, and passes the outputs to the symbol encoder **11** which encodes the frame symbols into amplitude and phase coded symbols **12** which could be complex $\{Z(u)\}$ or real $\{R(u_R),\ I(u_I)\}$ depending on the application. These symbols **12** are the inputs to the current real Walsh CDMA encoding in equations **(1)**. Inputs $\{Z(u)\}$, $\{R(u_R),\ I(u_I)\}$ **12** are Walsh encoded, summed over the users, and scrambled by PN in the current Walsh CDMA encoder **13** to generate the complex output chips $\{Z(n)\}$ **14**. This encoding **13** is a representative implementation of equations **(1)**. These output chips $Z(n)$ are waveform modulated **15** to generate the analog complex signal $z(t)$ which is single sideband upconverted, amplified, and transmitted (Tx) by the analog front end of the transmitter **15** as the real waveform $v(t)$ **16** at the carrier frequency $f_0$ whose amplitude is the real part of the complex envelope of the baseband waveform $z(t)$ multiplied by the carrier

frequency and the phase angle $\phi$ accounts for the phase change from the baseband signal to the transmitted signal.

FIG. **2** real Walsh CDMA encoding is a representative implementation of the Walsh CDMA encoding **13** in FIG. **1** and in equations **(1)**. Inputs are the user data symbols which could be complex $\{Z(u)\}$ or real $\{R(u_R), I(u_I)\}$ **17.** For complex and real data symbols the encoding of each user by the corresponding Walsh code is described in **18** by the implementation of transferring the sign of each Walsh code chip to the user data symbol followed by a 1-to-N expander $1\uparrow N$ of each data symbol into an N chip sequence using the sign transfer of the Walsh chips.

For complex data symbols $\{Z(u)\}$ the sign-expander operation **18** generates the N-chip sequence $Z(u,n) = Z(u)\mathrm{sgn}\{W(u,n)\} = Z(u)W(u,n)$ for $n=0,1,\ldots,N-1$ for each user $u=0,1,\ldots,N-1$. This Walsh encoding serves to spread each user data symbol into an orthogonally encoded chip sequence which is spread over the CDMA communications frequency band. The Walsh encoded chip sequences for each of the user data symbols are summed over the users **19** and encoded with a long code $P_2(n)$ followed by a short code $[P_R(n)+jP_I(n)]$ **21**. Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **22.** The switch **20** selects the appropriate signal processing path for the complex and real data symbols.

For real data symbols $\{R(u_R), jI(u_I)\}$ the real and imaginary communications axis data symbols are separately Walsh encoded **18,** summed **19,** and then PN encoded **19** with long codes $P_{R2}(n)$ for the real axis and $P_{I2}(n)$ for the imaginary axis to provide orthogonality between the channels along the real and imaginary communications axes. Output is complex combined **19** and PN encoded with the short PN sequence $[P_R(n)+ jP_I(n)]$ **21.** Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **22.**

FIG. **3** CDMA receiver block diagram is representative of a current CDMA receiver which includes an implementation of the current real Walsh CDMA decoding in equations **(2)**. This block diagram becomes a representative implementation of the CDMA receiver which implements the complex Walsh CDMA decoding when the current real Walsh CDMA decoding **27** is replaced by the complex Walsh CDMA decoding of this invention. FIG. **3** signal processing starts with the user transmitted wavefronts incident at the receiver antenna **23** for the $n_u$ users $u = 1, \ldots, n_u \leq N_c$. These wavefronts are combined by addition in the antenna to form the receive (Rx) signal $\hat{v}(t)$ at the antenna output **23** where $\hat{v}(t)$ is an estimate of the transmitted signal v(t) **16** in FIG. **1**, that is received with errors in time $\Delta t$, frequency $\Delta f$, phase $\Delta\theta$, and with an estimate $\hat{z}(t)$ of the transmitted complex baseband signal z(t) **16** in FIG. **1**. This received signal $\hat{v}(t)$ is amplified and downconverted by the analog front end **24** and then synchronized (synch.) and analog-to-digital (ADC) converted **25**. Outputs from the ADC are filtered and chip detected **26** by the fullband chip detector, to recover estimates $\{\hat{Z}(n) = \hat{R}(n) + j\,\hat{I}(n)\}$ **28** of the transmitted signal which is the stream of complex CDMA encoded chips $\{Z(n)=R(n)+jI(n)\}$ **14** in FIG. **1** for both complex and real data symbols. The CDMA decoder **27** implements the algorithms in equations **(2)** by stripping off the PN codes and decoding the received CDMA real Walsh orthogonally encoded chips to recover estimates $\{\hat{Z}(u) = \hat{R}(u_R) + j\hat{I}(u_I)\}$ **29** of the transmitted user data symbols $\{Z(u) = R(u_R) + jI(u_I)\}$ **12** in FIG. **1**. Notation introduced in FIG. **1** and **3** assumes that the user index $u=u_R=u_I$ for complex data symbols, and for real data symbols the user index u is used for counting the user pairs $(u_R, u_I)$ of real and complex data symbols. These estimates are processed by the symbol decoder **30** and the frame processor **31** to recover estimates **32** of the transmitted user data words.

FIG. **4** real Walsh CDMA decoding is a representative implementation of the real Walsh CDMA decoding **27** in FIG. **3** and in equations **(2)**. Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **33**. The PN codes are stripped off from these chips **34** by multiplying by the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per the decoding algorithms **7** in equations **(2)**.

For complex data symbols **35** the long PN code is stripped off and the real Walsh channelization coding is removed by a pulse compression operation consisting of multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user, scaling by 1/2N, and summing the products over the N Walsh chips **36** to recover estimates $\{\hat{Z}(u)\}$ of the user complex data symbols $\{Z(u)\}$. The switch **35** selects the appropriate signal processing path for the complex and real data symbols.

For real data symbols **35** the next signal processing operation is the removal of the remaining PN codes from the real and imaginary axes. This is followed by stripping off the Walsh channelization coding by multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user, scaling by 1/2N, and summing the products over the N Walsh chips **36** to recover estimates $\{\hat{R}(u_R),\ \hat{I}(u_I)\}$ of the user real data symbols $\{R(u_R),\ I(u_I)\}$.

It should be obvious to anyone skilled in the communications art that these example implementations clearly define the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that these examples are representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable of this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability of this invention.

## SUMMARY OF THE INVENTION

This invention is a new approach to the application of Walsh orthogonal codes for CDMA, which offers to replaces the current real Walsh codes with complex Walsh codes called hybrid Walsh codes and generalized complex Walsh codes called generalized hybrid Walsh codes. Real Walsh codes are used for current CDMA applications and complex Walsh codes will provide the choice of using complex Walsh codes or the real Walsh codes since the permutated real Walsh codes are the real components of the complex Walsh codes. This means an application capable of using the complex Walsh codes can simply turn-off the complex axis components of the complex Walsh codes for real Walsh CDMA coding and decoding.

The complex Walsh codes of this invention are proven to be the natural development for the Walsh codes and therefore are the correct complex Walsh codes to within arbitrary factors that include scale and rotation, which are not relevant to performance. This natural development of the complex Walsh codes in the N-dimensional complex code space $C^N$ extended the correspondences between the real Walsh codes and the Fourier codes in the N-dimensional real code space $R^N$, to correspondences between the complex Walsh codes and the discrete Fourier transform (DFT) complex codes in $C^N$.

These 4-phase complex Walsh orthogonal CDMA codes provide fundamental performance improvements compared to the 2-phase real Walsh codes which include an increase in the carrier-to-noise ratio (CNR) for data symbol recovery in the receiver, lower correlation side-lobes under timing offsets both with and without PN spreading, lower levels of harmonic interference caused by non-linear amplification of multi-carrier CDMA signals, and reduced phase tracking jitter for code tracking to support both acquisition and synchronization. These potential performace improvements simply reflect the widely known principle that complex CDMA is better than real CDMA.

The gemeralized complex Walsh orthogonal CDMA codes increase the choices for the code length by allowing the combined use of hybrid Walsh, Walsh, and discrete Fourier transform complex orthogonal codes using a Kronecker or tensor cconstruction, direct sum construction, as well as the possibility for more general functional combining.

## BRIEF DESCRIPTION OF THE DRAWINGS AND THE PERFORMANCE DATA

The above-mentioned and other features, objects, design algorithms, implementations, and performance advantages of the present invention will become more apparent from the detailed description set forth below when taken in conjunction with the drawings and performance data wherein like reference characters and numerals denote like elements, and in which:

FIG. **1** is a representative CDMA transmitter signal processing implementation block diagram with emphasis on the

current real Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **2** is a representative CDMA encoding signal processing implementation diagram with emphasis on the current real Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **3** is a representative CDMA receiver signal processing implementation block diagram with emphasis on the current real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **4** is a representative CDMA decoding signal processing implementation diagram with emphasis on the current real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. **5** is a representative correlation plot of the correlation between the complex discrete Fourier transform (DFT) cosine and sine code component vectors and the real Fourier transform cosine and sine code component vectors.

FIG. **6** is a representative CDMA encoding signal processing implementation diagram with emphasis on the hybrid Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure

FIG. **7** is a representative CDMA decoding signal processing implementation diagram with emphasis on the hybrid Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

## DISCLOSURE OF THE INVENTION

Consider the real orthogonal CDMA code space $R^N$ for Hadamard, Walsh, and Fourier codes. The new complex Walsh orthogonal CDMA codes are called hybrid Walsh codes and are derived from the current real Walsh codes by starting with the correspondence of the current real Walsh codes with the discrete real Fourier transform basis vectors. Examples of code sets in $R^N$ consisting of N-orthogonal real code vectors include the Hadamard, Walsh, and Fourier. The corresponding matrices of code vectors are designated as H, W, F respectively and as defined in equation **(3)** consist of N-rows of N-chip code vectors. Hadamard codes in their re-ordered form known as Walsh codes are used in the current CDMA, in the proposals for the next generation G3 CDMA, and in the proposals for all future CDMA. Walsh codes re-order the Hadamard codes according to increasing sequency. Sequency is the average rate of change of the sign of the codes.

Equation **(3)** define the three sets H,W,F of real orthogonal codes in $R^N$ with the understanding that the H and W are identical except for the ordering of the code vectors. Hadamard **37** and Walsh **38** orthogonal functions are basis vectors in $R^N$ and are used as code vectors for orthogonal CDMA channelization coding. Hadamard **37** and Walsh **38** equations of definition are widely known. Likewise, the Fourier **39** equations of definition are widely known within the engineering and scientific communities, wherein

N-chip real orthogonal CDMA codes **(3)**

**37** Hadamard codes

H = Hadamard NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [ H(u) ] matrix of row vectors H(u)

= [ H(u,n) ] matrix of elements H(u,n)

H(u) = Hadamard code vector u

= [ H(u,0), H(u,1), …, H(u,N-1) ]

= 1xN row vector of chips H(u,0),….,H(u,N-1)

H(u,n) = Hadamard code u chip n

= +/+1 possible values

$$= (-1)\hat{}\ [\ \sum_{i=0}^{i=M-1} u_i n_i\ ]$$

where $u = \sum_{i=0}^{i=M-1} \mathbf{u}_i 2^i$ binary representation of u

$n = \sum_{i=0}^{i=M-1} n_i 2^i$ binary representation of n

**38** Walsh codes

W = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [ W(u) ] matrix of row vectors W(u)

= [ W(u,n) ] matrix of elements W(u,n)

W(u) =Walsh code vector u

= [ W(u,0), W(u,1), …, W(u,N-1) ]

W(u,n) = Walsh code u chip n

= +/+1 possible values

$$= (-1)\hat{}\ [u_{M-1}n_0 + \sum_{i=1}^{i=M-1} (u_{M-1-i} + u_{M-i})n_i\ ]$$

**39** Fourier codes

F = Fourier NxN orthogonal code matrix consisting of N rows of N chip code vectors

$$= [\ F(u)\ ] \text{ matrix of row vectors } F(u)$$

$$= \left[\frac{C}{S}\right]$$

$$C = N/2+1 \text{ x } N \text{ matrix of row vectors } C(u)$$

$$C(u) = \text{Even code vectors for } u=0,1,\ldots,N/2$$

$$= [1,\ \cos(2\pi\ u/N),\ldots,\cos(2\pi\ u/2)]$$

$$S = N/2-1 \text{ x } N \text{ matrix of row vectors } S(u)$$

$$S(\Delta u) = \text{Odd code vectors for } u=N/2+\Delta u,\ \Delta u=1,2,\ldots,N/2-1$$

$$= [\sin(2\pi\ \Delta u/N),\ldots,\sin(2\pi\ \Delta u(1/2-1/N))]$$

$$\text{where } F(u) = C(u) \quad \text{for } u=0,1,\ldots,N/2$$

$$= S(\Delta u) \quad \text{for } \Delta u = u-N/2,\ u=N/2+1,\ldots,N-1$$

the cosine C(u) and sine S(u) code vectors are the code vectors of the Fourier code matrix F.

Consider the complex orthogonal CDMA code space $C^N$ for DFT codes. The DFT orthogonal codes are a complex basis for the complex N-dimensional CDMA code space $C^N$ and consist of the DFT harmonic code vectors arranged in increasing order of frequency. Equations **(4)** are the definition of the DFT code vectors. The DFT definition **40** is widely known within the engineering and scientific communities. Even and odd components of the DFT code vectors **41** are the real cosine code vectors {C(u)} and the imaginary sine code vectors {S(u)} where even and odd are referenced to the midpoint of the code vectors. These cosine and sine code vectors are the extended set 2N of the N Fourier cosine and sine code vectors.

N-chip DFT complex orthogonal CDMA codes **(4)**

**40** DFT code vectors

E = DFT NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [ E(u) ] matrix of row vectors E(u)

= [ E(u,n) ] matrix of elements E(u,n)

E(u) = DFT code vector u

= [ E(u,0), E(u,1), …, E(u,N-1) ]

= 1xN row vector of chips E(u,0),….,E(u,N-1)

E(u,n) = DFT code u chip n

= $e^{j2\pi\, un/N}$

= $\cos(2\pi\, un/N) + j\sin(2\pi\, un/N)$

= N possible values on the unit circle

**41** Even and odd code vectors are the extended set of Fourier even and odd code vectors in **39** equations **(3)**

C(u) = Even code vectors for u=0,1,…,N-1

= $[1, \cos(2\pi\, u\, 1/N),\ldots,\cos(2\pi\, u\, (N-1)/N)]$

S(u) = Odd code vectors for u=0,1,…,N-1

= $[0, \sin(2\pi\, u\, 1/N),\ldots,\sin(2\pi\, u\, (N-1)/N)]$

E(u) = C(u) + j S(u) for u=0,1,…,N-1

Consider the complex orthogonal CDMA code space $C^N$ for hybrid Walsh codes. Step **1** in the derivation of the hybrid Walsh codes in this invention establishes the correspondence of the even and odd Walsh codes with the even and odd Fourier codes. Even and odd for these codes are with respect to the midpoint of the row vectors similar to the definition for the DFT vector codes **41** in equations **(4)**. Equations **(5)** identify the even and odd Walsh codes in the W basis in $R^N$. These even and odd Walsh codes can be placed in

Even and odd Walsh codes in $R^N$ **(5)**

$W_e(u)$ = Even Walsh code vector

= $W(2u)$ for $u=0,1,\ldots,N/2-1$

$W_o(u)$ = Odd Walsh code vectors

= $W(2u-1)$ for $u=1,\ldots,N/2$

direct correspondence with the Fourier code vectors **39** in equations **(3)** using the DFT equations **(4)**. This correspondence is defined in equations **(6)** where the correspondence operator "~" represents the even and odd correspondence between the Walsh and Fourier codes, and additionally represents the sequency~frequency correspondence.

Correspondence between Walsh and Fourier codes **(6)**

$W(0) \sim C(0)$

$W_e(u) \sim C(u)$ for $u=1,\ldots,N/2-1$

$W_o(u) \sim S(u)$ for $u=1,\ldots,N/2-1$

$W(N-1) \sim C(N/2)$

Step **2** derives the set of N complex DFT vector codes in $C^N$ from the set of N real Fourier vector codes in $R^N$. This means that the set of 2N cosine and sine code vectors in **41** in equations **(4)** for the DFT codes in $C^N$ will be derived from the set of N cosine and sine code vectors in **39** in equations **(3)** for the Fourier codes in $R^N$. The first N/2+1 code vectors of the DFT basis can be written in terms of the Fourier code vectors in equations **(7)**.

DFT code vectors 0,1,…,N/2 derived from Fourier **(7)**

**42** Fourier code vectors from **39** in equations **(3)** are

C(u) = Even code vectors for u=0,1,…,N/2

= [1, cos(2$\pi$ u1/N),…,cos(2$\pi$ u/2)]

S(u) = Odd code vectors for u=1,2,…,N/2-1

= [sin(2$\pi$ u1/N),…,sin(2$\pi$ u (1/2-1/N))]

**43** DFT code vectors in **41** of equations **(4)** are written as functions of the Fourier code vectors

E(u) = DFT complex code vectors for u=0,1,…,N/2

= C(0)

= C(u)+jS(u) for u=1,…,N/2-1

= C(N/2) for u=N/2

The remaining set of N/2+1,…,N-1 DFT code vectors in $C^N$ can be derived from the original set of Fourier code vectors by a correlation which establishes the mapping of the DFT codes onto the Fourier codes. We derive this mapping by correlating the real and imaginary components of the DFT code vectors with the corresponding even and odd components of the Fourier code vectors. The correlation operation is defined in equations **(8)**

Correlation of DFT and Fourier code vectors **(8)**

Corr(even) = C*Real{E′}

= Correlation matrix

= Matrix product of C and the real part of E transpose

Corr(odd) = S*Imag{E′}

= Correlation matrix

= Matrix product of S and the imaginary part of E transpose

wherein "*" is the matrix product, "′" is the conjugate transpose operator, and the results of the correlation calculations are plotted in FIG. **5** for N=32 for the real cosine and the odd sine Fourier code vectors. Plotted are the correlation of the 2N DFT cosine and sine codes against the N Fourier cosine and sine codes which range from -15 to +16 where the negative indices of the codes represent a negative correlation value. The plotted curves are the correlation peaks. These correlation curves in FIG. **5** prove that the remaining N/2+1,…,N-1 code vectors of the DFT are derived from the Fourier code vectors by equations **(9)**. This

DFT code vectors N/2+1,…, N-1 derived from Fourier **(9)**

$$E(u) = C(N/2 - \Delta u) - jS(N/2 - \Delta u)$$

$$\text{for} \quad u = N/2 + \Delta u$$

$$\Delta u = 1,\ldots,N/2-1$$

construction of the remaining DFT basis in equations **(9)** is an application of the DFT spectral foldover property which observes the DFT harmonic vectors for frequencies $fNT=N/2+\Delta u$ above the Nyquist sampling rate $fNT=N/2$ simply foldover such that the DFT harmonic vector for $fNT = N/2+\Delta u$ is the DFT basis vector for $fNT = N/2-\Delta u$ to within a fixed sign and where "f" is frequency and "T" is sample interval.

Step **3** derives the hybrid Walsh code vectors from the real Walsh code vectors by using the DFT derivation in equations **(7)** and **(9)**, by using the correspondences between the real Walsh and Fourier in equations **(6)**, and by using the fundamental correspondence between the hybrid Walsh and the complex DFT given in equation **(10)**.

Correspondence between hybrid Walsh and DFT **(10)**

$\tilde{W} \sim E$ = NxN complex DFT orthogonal code matrix

where

$\tilde{W}$ = NxN hybrid Walsh orthogonal code matrix

= N rows of N chip code vectors

= $[\tilde{W}(u)]$ matrix of row vectors $\tilde{W}(u)$

= $[\tilde{W}(u,n)]$ matrix of elements $\tilde{W}(u,n)$

$\tilde{W}(u)$ = Hybrid Walsh code vector u

= $[\tilde{W}(u,0), \tilde{W}(u,1), \ldots, \tilde{W}(u,N-1)]$

$\tilde{W}$ = +/-1 +/- j possible value

We start by constructing the hybrid Walsh dc code vector $\tilde{W}(0)$. We use equation $E(0)=C(0)$ in **43** in equations **(7)**, the correspondence in equations **(6)**, and observe that the dc hybrid Walsh vector has both real and imaginary components in the $\tilde{W}$ domain, to derive the dc hybrid Walsh code vector equation:

$$\tilde{W}(0) = W(0) + jW(0) \qquad \text{for } u=0 \tag{11}$$

For hybrid Walsh code vectors $\tilde{W}(u)$, u=1,2,…,N/2-1, we start with the Walsh code properties in **(5)**,**(6)** and apply the correspondences in equations **(10)** between the hybrid Walsh and DFT bases, to the DFT equations **43** in equations **(7)** to derive the equations:

$$\widetilde{W}(u) = W_e(u) + jW_o(u) \qquad \text{for } u=1,2,\ldots,N/2-1 \qquad \textbf{(12)}$$
$$= W(2u) + jW(2u-1) \qquad \text{for } u=1,2,\ldots,N/2-1$$

For hybrid Walsh code vector $\widetilde{W}(N/2)$ we use the equation $E(N/2)=C(N/2)$ **43** in equations **(7)** and the same rationale used to derive equation **(11)**, to derive the equation:

$$\widetilde{W}(u) = W(N-1) + jW(N-1) \qquad \text{for } u=N/2 \qquad \textbf{(13)}$$

For hybrid Walsh code vectors $\widetilde{W}(N/2+\Delta u)$, $\Delta u=1,2,\ldots,N/2-1$ we apply the correspondences between the hybrid Walsh and DFT bases to the spectral foldover equation $E(N/2+\Delta u)=C(N/2-\Delta u)-jS(N/2-\Delta u)$ in equation **(9)** with the changes in indexing required to account for the W indexing in equations **(5)**,**(6)** to derive the equations:

$$\widetilde{W}(N/2+\Delta u) = W(N-1-\Delta eu)+W(N-1-\Delta ou) \qquad \textbf{(14)}$$
$$\text{for } u=N/2+1,\ldots,N-1$$
$$= W(N-1-2\Delta u) + jW(N-2\Delta u)$$
$$\text{for } u=N/2+1,\ldots,N-1$$

using the notation $\Delta eu=2\Delta u$, $\Delta ou=2\Delta u-1$. These hyubrid Walsh code vectors in equations **(11)**, **(12)**, **(13)**, **(14)** are the equations of definition for the hybrid Walsh code vectors.

An equivalent way to derive the hybrid Walsh code vectors in $C^N$ from the real Walsh basis in $R^{2N}$ is to use a sampling technique which is a known method for deriving a complex basis in $C^N$ from a real basis in $R^{2N}$.

Transmitter equations **(15)** describe a representative hybrid Walsh CDMA encoding for the transmitter in FIG. **1** It is assumed that there are N hybrid Walsh code vectors $\widetilde{W}(u)$ **44** which are the 1xN row vectors of the NxN hybrid Walsh matrix $\widetilde{W}$. Each user is assigned a unique Walsh code which allows the code vectors to be designated by the user symbols u=0,1,…,N-1 for N hybrid Walsh codes. The hybrid Walsh code vectors $\widetilde{W}(u)$ derived in equations **(11)**,**(12)**,**(13)**,**(14)** are summarized **44** in terms of their real and imaginary component code vectors $\widetilde{W}(u)=W_R(u)+jW_I(u)$ where $W_R(u)$ and $W_I(u)$ are respectively the real and imaginary component code vectors. As per the derivation of $\widetilde{W}(u)$ the sets of real axis code vectors $\{W_R(u)\}$ and the imaginary axis code vectors $\{W_I(u)\}$ both consist of the Walsh code vectors in $R^N$ with the ordering modified to ensure that the definition of the hybrid Walsh vectors satisfies equations **(11)**, **(12)**, **(13)**, **(14)**.

Hybrid Walsh CDMA encoding for transmitter **(15)**

**44** Hybrid Walsh codes defined in **(11)**,**(12)**,**(13)**,**(14)**

$$\begin{aligned}\widetilde{W}(u) &= \text{Hybrid Walsh code vector } u \\ &= W_R(u) + jW_I(u) \qquad \text{for } u=0,1,\ldots,N-1\end{aligned}$$

where

$$\begin{aligned}W_R(u) &= \text{Real}\{\ \widetilde{W}(u)\ \} \\ &= W(0) && \text{for } u=0 \\ &= W(2u) && \text{for } u=1,2,\ldots,N/2-1 \\ &= W(N-1) && \text{for } u=N/2 \\ &= W(2N-2u-1) && \text{for } u=N/2+1,\ldots,N-1\end{aligned}$$

$$
\begin{aligned}
W_I(u) &= \mathrm{Imag}\{\ \tilde{W}(u)\ \} \\
&= W(0) && \text{for } u=0 \\
&= W(2u-1) && \text{for } u=1,2,\ldots,N/2-1 \\
&= W(N-1) && \text{for } u=N/2 \\
&= W(2N-2u) && \text{for } u=N/2+1,\ldots,N-1
\end{aligned}
$$

**45** Data symbols

$$
\begin{aligned}
Z(u) &= \text{Complex data symbol for user } u \\
&= R(u) + jI(u)
\end{aligned}
$$

**46** Hybrid Walsh encoded data

$$
\begin{aligned}
Z(u,n) &= Z(u)\ \tilde{W}(u,n) \\
&= Z(u)[\mathrm{sgn}\{W_R(u,n)\} + j\mathrm{sgn}\{W_I(u,n)\}] \\
&= [R(u)\mathrm{sgn}\{W_R(u,n)\} - I(u)\mathrm{sgn}\{W_I(u,n)\}] \\
&\quad + j[R(u)\mathrm{sgn}\{W_I(u,n) + I(u,n)\mathrm{sgn}\{W_R(u,n)\}]
\end{aligned}
$$

**47** PN scrambling

$$
\begin{aligned}
Z(n) &= \text{PN scrambled complex Walsh encoded data chips after summing over the users} \\
&= \sum_u Z(u,n)\,P_2(n)\,[P_R(n) + j\,P_I(n)] \\
&= \sum_u Z(u,n)\,\mathrm{sgn}\{P_2(n)\}\,[\mathrm{sgn}\{P_R(n)\} + j\,\mathrm{sgn}\{P_I(n)\}] \\
&= \text{Hybrid Walsh CDMA encoded chips}
\end{aligned}
$$

User data symbols **45** are the set of complex symbols $\{Z(u), u=0,1\ldots,N-1\}$. These data symbols are encoded by the hybrid Walsh CDMA codes **46**, encoded with PN scrambling codes comprising both long and short codes **47**, and summed over the users to yield the hybrid Walsh CDMA encoded chips $Z(n)$. Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in equations **(1)**.

Receiver equations **(16)** describe a representative hybrid Walsh CDMA decoding for the receiver in FIG. **3**. The receiver front end **48** provides estimates $\{\hat{Z}(n)\}$ of the transmitted Walsh CDMA encoded chips {Z(n)} for the complex data symbols {Z(u)}. Orthogonality property **49** is expressed as a matrix product of the hybrid Walsh code chips or equivalently as a matrix of the hybrid Walsh code chip numerical signs of the real and imaginary components. Decoding algorithms **51** perform the inverse of the signal processing for the encoding in equations **(15)** to recover estimates $\{\hat{Z}(u)\}$ of the transmitter user symbols {Z(n)} for the complex data symbols {Z(u)}. Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in equations **(2)**.

Hybrid Walsh CDMA decoding for receiver **(16)**

**48** Receiver front end in FIG. **3** provides estimates $\{\hat{Z}(n)\}$ **28** of the encoded transmitter chip symbols {Z(n)} **47** in equations **(15)**.

**49** Orthogoality property of Walsh NxN matrix $\widetilde{W}$

$$\sum_{\mathbf{n}} \widetilde{\mathbf{W}}(\hat{\mathbf{u}},\mathbf{n})\,\widetilde{\mathbf{W}}'(\mathbf{n},\mathbf{u}) =$$

$$\sum_n [\operatorname{sgn}\{W_R(\hat{u},n)\} + j\operatorname{sgn}\{W_I(\hat{u},n)\}][\operatorname{sgn}\{W_R(n,u)\} - j\operatorname{sgn}\{W_I(n,u)\}]$$

$$= 2N\ \delta(\hat{u}, u)$$

where $\delta(\hat{u}, u)$ = Delta function of $\hat{u}$ and u

= 1 for $\hat{u} =$ u

= 0 otherwise

**51** Decoding algorithm

$$\hat{Z}(u) = 4^{-1}N^{-1} \sum_n \hat{Z}(n)\, sgn\{P_2(n)\}\, [sgn\{P_R(n)\} - j\, sgn\{P_I(n)\}]^* = [sgn\{W_R(n,u)\} - j\, sgn\{W_I(n,u)\}]$$

= Receiver estimate of the transmitted data symbol Z(u) **45** in equations **(15)**

FIG. **6** hybrid Walsh CDMA encoding is a representative implementation of the hybrid Walsh CDMA encoding which will replace the current real Walsh encoding **13** in FIG. **1** and is defined in equations **(15)**. Inputs are the user data symbols {Z(u)} **52**. Encoding of each user by the corresponding hybrid Walsh code implements the hybrid Walsh encoding in **46** in equations **(15)**. Encoded chip sequences for each of the user data symbols are summed over the users **54** followed by PN encoding with long and short codes **55**. Output is the stream of complex CDMA encoded chips {Z(n)} **56**. Combinations of both real and complex data symbols can be used similar to the approach for the real Walsh in FIG. **2**.

FIG. **7** hybrid Walsh CDMA decoding is a representative implementation of hybrid Walsh CDMA decoding which will replace the current real Walsh decoding **27** in FIG. **3**, and is defined in equations **(16)**. Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **57**. The PN scrambling code is stripped off from these chips **58** using the decoding algorithm in equations **(16)**. The hybrid Walsh channelization coding is removed **59** and the output scaled by 1/4N to recover estimates $\{\hat{Z}(u)\}$ of the user complex data symbols {Z(u)}. Combinations of both

real and complex data symbols can be used similar to the approach for the real Walsh in FIG. **4**.

It should be obvious to anyone skilled in the communications art that this example implementations in FIG. 6, **7** clearly define the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

Consider complex orthogonal CDMA code space $C^N$ for generalized hybrid Walsh codes which allow the code length N to be a product of powers of primes **60** in equations **(17)** or a sum of powers of primes **61** in equations **(17)**, at the implementation cost of introducing multiply operations into the CDMA encoding and decoding. In the previous disclosure of this invention the N was assumed to be equal to a power of 2 which means N=2^m corresponding to prime $p_0=2$ and integer $M=m_0$. This restriction was made for convenience in explaining the construction of the hybrid Walsh and is not required since it is well known that Hadamard matrices exist for non-integer powers of 2 and, therefore, hybrid Walsh matrices exist for non-integer powers of 2.

Length N of generalized hybrid Walsh codes **(17)**

**60** Kronecker or tensor product code construction

$$N = \prod_k p_k\hat{\ }m_k$$

$$= \prod_k N_k$$

where

$p_k$ = prime number indexed by k starting with k=0

$m_k$ = order of the prime number $p_k$

$N_k$ = Length of code for the prime $p_k$

= $p_k\hat{\ }m_k$

**61** Direct sum code construction

$$N = \sum_k p_k\hat{\ }m_k$$

$$= \sum_k N_k$$

Add-only arithmetic operations are required for encoding and decoding both real Walsh and hybrid Walsh CDMA codes since the real Walsh values are +/-1 and the hybrid Walsh values are {+/-1 +/-j} or equivalently are {1,j,-1,-j} under a -90 degree rotation and normalization which means the only operations are sign transfers and adds plus subtracts add-only algebraic operations. Multiply operations are more complex to implement than add operations. However, the advantages of having greater flexibility in choosing the orthogonal CDMA code lengths N using equations **(17)** can offset the expense of multiply operations for particular applications. Accordingly, this invention includes the concept of generalized hybrid Walsh orthogonal CDMA codes with the flexibility to meet these needs. This extended class of hybrid Walsh codes supplements the hybrid Walsh codes by combining with Hadamard, real Walsh, DFT, and other orthogonal codes as well as with quasi-orthogonal PN by relaxing the orthogonality property to quasi-orthogonality.

Generalized hybrid Walsh orthogonal CDMA codes can be constructed as demonstrated in **64** and **65** in equations **(18)** for the Kronecker or tensor product, and in **66** for the direct sum. The example code matrices considered for orthogonal CDMA codes in **62** for the construction of the generalized hybrid Walsh are the DFT E and Hadamard H or equivalently Walsh W, in addition to the hybrid Walsh $\widetilde{W}$. The algorithms and examples for the construction start with the definitions **63** of the NxN orthogonal code matrices $\widetilde{W}=\widetilde{W}_N$ , $E=E_N$, $H=H_N$ for N=2,4. and the equivalence of $E_4$ and $\widetilde{W}_4$ after the $\widetilde{W}_4$ is rotated through the angle -45 degrees and rescaled. The CDMA current and developing standards use the prime 2 which generates a code length $N=2^M$ where M=integer. For applications requiring greater flexibility in code length N, additional primes can be used using the tensor construction. This flexibility is illustrated in **65** with the addition of prime=3. The use of prime=3 in addition to the prime=2 in the range of N=8 to N=64 is observed to increase the number of N choices at a modest cost penalty of using multiples of the angle increment 30 degrees for prime=3 in addition to the angle increment 90 degrees for prime=2. As noted in **65** there are several choices in the ordering of the tensor product construction and 2 of these choices are used in the construction and these choices yield different sets of orthogonal codes. Direct sum construction provides greater flexibility in the choice of N without necessarily introducing a multiply penality. However, the addition of the zero matrix in the construction is generally not desirable for CDMA communications.

**62** Code matrices

$\widetilde{W}_N$ = NxN hybrid Walsh code matrix

$E_N$ = NxN DFT code matrix

$H_N$ = NxN Hadamard code matrix

**63** Low-order code definitions and equivalences

$$2\text{x}2 \quad H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= E_2$$

$$= (e^{-j\pi/4}/\sqrt{2}) * \widetilde{W}_2$$

$$3\text{x}3 \quad E_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j2\pi/3} & e^{j2\pi 2/3} \\ 1 & e^{j2\pi 2/3} & e^{j2\pi/3} \end{bmatrix}$$

$$4\text{x}4 \quad H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\widetilde{W}_4 = \begin{bmatrix} 1+j & 1+j & 1+j & 1+j \\ 1+j & -1+j & -1-j & 1-j \\ 1+j & -1-j & 1+j & -1-j \\ 1+j & 1-j & -1-j & -1+j \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$= (e^{-j\pi/4}/\sqrt{2})\ \widetilde{W}_4$$

**64** Kronecker or tensor product construction for $N=\prod_k N_k$

Code matrix $C_N$ = NxN generalized hybrid Walsh CDMA code matrix

Kronecker or tensor product construction of $C_N$

$$C_N = C_0 \prod_{k>0} \otimes C_{N_k}$$

Kronecker or tensor product definition

$A = N_a x N_a$ orthogonal code matrix $[a_{ik}]$

$B = N_b x N_b$ orthogonal code matrix

$A \otimes B$ = Kronecker or tensor product of matrix A and matrix B

= $N_a N_b$ x $N_a N_b$ orthogonal code matrix consisting of the elements $[a_{ik}]$ of matrix A multiplied by the matrix B

= $[\ a_{ik}\ B\ ]$

**65** Kronecker or tensor product construction examples for primes p=2,3 and the range of sizes $8 \le N \le 64$

8x8 $C_8 = \widetilde{\mathbf{W}}_8$

12x12 $C_{12} = \widetilde{W}_4 \otimes E_3$

$C_{12} = E_3 \otimes \widetilde{W}_4$

16x16 $C_{16} = \widetilde{W}_{16}$

18x18 $C_{18} = \widetilde{W}_2 \otimes E_3 \otimes E_3$

$C_{18} = E_3 \otimes E_3 \otimes \widetilde{W}_2$

24x24 $C_{24} = \widetilde{W}_8 \otimes E_3$

$C_{24} = E_3 \otimes \widetilde{W}_8$

32x32 $C_{32} = \widetilde{W}_{32}$

36x36 $C_{36} = \widetilde{W}_4 \otimes \widetilde{W}_3 \otimes \widetilde{W}_3$

$C_{36} = \widetilde{W}_3 \otimes \widetilde{W}_3 \otimes \widetilde{W}_4$

48x48 $C_{48} = \widetilde{W}_{16} \otimes \widetilde{W}_3$

$C_{48} = \widetilde{W}_3 \otimes \widetilde{W}_{16}$

64x64 $C_{64} = \widetilde{W}_{64}$

**66** Direct sum construction for $N=\sum_k N_k$

Code matrix $C_N$ = NxN hybrid orthogonal CDMA code matrix

Direct sum construction of $C_N$

$$C_N = C_0 \prod_{k>0} \oplus C_{N_k}$$

Direct sum definition

A = $N_a x N_a$ orthogonal code matrix

B = $N_b x N_b$ orthogonal code matrix

$A \oplus B$ = Direct sum of matrix A and matrix B

= $N_a+N_b$ x $N_a+N_b$ orthogonal code matrix

$$= \left[\begin{array}{c|c} A & O_{N_a x N_b} \\ \hline O_{N_b x N_a} & B \end{array}\right]$$

where $O_{N_1 x N_2} = N_1 x N_2$ zero matrix

It should be obvious to anyone skilled in the communications art that these example implementations of the generalized hybrid Walsh in equations **(18)** clearly define the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches. For example, the Kronecker or tensor product matrices $E_N$ and $H_N$ can be replaced by functionals.

For cellular applications the transmitter description which includes equations **(18)** describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver corresponding to the decoding of equations **(18)** describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

Consider computationally efficient encoding and decoding of complex Walsh CDMA codes and hybrid complex Walsh CDMA codes. It is well known that fast and efficient encoding and decoding algorithms exist for the real Walsh CDMA codes. It is obvious that with suitable modifications these algorithms can be used to develop fast and efficient encoding and decoding algorithms for the hybrid Walsh CDMA codes since these complex codes have real and imaginary code vectors which are from the same set of real Walsh CDMA codes.

It is well known that the Kronecker or tensor product construction involving DFT, H and real Walsh orthogonal code vectors have efficient encoding and decoding algorithms. It is obvious that with suitable modifications these algorithms can be used to develop fast and efficient encoding and decoding algorithms for the Kronecker or tensor products of DFT, H and hybrid Walsh CDMA codes since these hybrid Walsh codes have real and imaginary code vectors which are from the same set of real Walsh CDMA codes. It is obvious that fast and efficient encoding and decoding algorithms exist for direct sum construction and functional combining.

Preferred embodiments in the previous description is provided to enable any person skilled in the art to make or use the present invention. The various modifications to these embodiments will be readily apparent to those skilled in the art, and the generic principles defined herein may be applied to other embodiments without the use of the inventive faculty. Thus, the present invention is not intended to be limited to the embodiments shown herein but is to be accorded the wider scope consistent with the principles and novel features disclosed herein.

APPLICATION NO. 09/826,117

TITLE OF INVENTION: Hyhrid Walsh Codes for CDMA

INVENTOR: Urbain A. von der Embse

Currently amended DRAWINGS AND PERFORMANCE DATA